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Towards the Compilation of the Global Twilight Pattern

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Abstract

This paper summarizes ISRN's 3-years research on the occurrence of dawn and dusk to mark the timings of the Fajr and Isha prayers. We acquired the astronomical data not only in Indonesia but also in Malaysia, the US, Egypt, and Turkey. It will be an attempt to compile a global twilight pattern in the future. The main instrument used is a Sky Quality Meter (SQM) that records the sky brightness data. For quality assurance, ISRN-UHAMKA employed dozens of imaging sensors ranging from an All Sky Camera, different types of DSLR, and gadget cameras. From hundreds of astronomical data, we have collected; it seems that the occurrence of the real twilight to mark the Fajr prayer throughout the world is the same. That is when the sun is at its depression angle of around 12^0-13^0. Likewise, the dusk to mark the end of the Maghrib prayer also occurs when the sun is at a solar depression angle around 12^0-13^0.

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Meanwhile, Muslims in the world use different solar depression angles ranging from -15^0 to -20^0.fields.

Keywords: *global twilight pattern, sun depression angle, sky brightness, poly-nomial regression model, image processing and analysis*

1. Introduction

Dawn (Arabic: Fajr) has long been the interest of the world's inhabitants as it marks the beginning of a day whereby creatures have to start their lives in seek- ing the bounty provided by their creator. For Muslims, it even more emphasizes as the moment also marks the first moment of submission before the Almighty by performing the Fajr prayer. On the other end of the day, Muslims also have to perform Isha prayer when the evening twilight ends. Astronomically, although the sun is still below the horizon at certain dip (sun depression angle), the sun- light has started lightening the sky. It happens because the atmosphere layers around the earth have reflected and spread the sunlight onto the sky. On the contrary, although the sun has already set in the western horizon, the sky does not become dark immediately. The sky is gradually becoming darker until it reaches its total dark (dusk) when the atmosphere is not able to spread the ever-deeper sun below the horizon. For this, the government of Indonesia has long set that the dawn occurs when the sun is at its depression angle of 20^0 below the east- ern horizon, while the dusk appears when the sun is at the dip of 18^0 below the western horizon.

1.1 Initial Objections

Curiosity, however, has sparked among Muslim's clerics that the above dips are far too deep. In other words, when Indonesian Muslims perform their Fajr prayer, it is much too early. While the timing for Isha is just too late, which might cause Muslims to perform Maghrib prayer too late. In Islam, the however perfect one is performing the prayer, it is meaningless if he/she is not in the right time prescribed by the Al Quran and Hadith.

It emerged for the first time when a rather limited circulated Islamic magazine, Qiblati, raised the issue, back in 2009. However, the controversy just faded away as there was no or too little scientific evidence to support the claim. This issue reemerged in 2016, when the largest Muslims organization, Muhammadiyah, organized a closed discussion regarding the premature dawn. Three researchers from Institut Teknologi Bandung (ITB), Universitas Ahmad Dahlan (UAD) Yogyakarta, and Universitas Muhammadiyah Prof. Dr. HAMKA (UHAMKA) Jakarta presented their early findings. As a research institute under the jurisdiction of Muhammadiyah as a big umbrella, the Islamic Science Re- search Network (ISRN) - UHAMKA chosed this topic as core research in the following years. In March 2017, we commenced our comprehensive research by using the Sky Quality Meter (SQM) as our main instrument that is widely used by environmentalists.

The unit to express the sky brightness is the magnitude. Positive magnitude indicates a dark sky, while negative magnitude indicates a bright sky. An SQM can automatically record the sky brightness data continuously. Usually, ISRN records them at a temporal resolution of threeseconds. Mathematically, the abrupt change of the trend of the data as shown by the arrows in Figure 1 mark the occurrence of dawn and dusk.

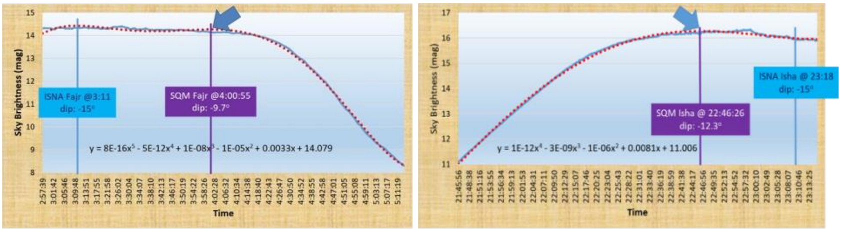

Figure 1: The official and real Fajr in Tacoma on June 18, 2019 (left), real and official Isha in Tacoma on Jun 17, 2019 (right).

There are at least two verses in the Koran, which precisely match with graphical location of the arrow in the SQM data plot in Figure 1. The first one is Al-Baqarah: 187, which defines the beginning of the Fajr prayer. It reads:

وَكُلُواْ وَاشْرَبُواْ حَتَّى يَتَبَيَّنَ لَكُمُ الْخَيْطُ الأَبْيَضُ مِنَ الْخَيْطِ الأَسْوَدِ مِنَ الْفَجْرِ

And eat and drink until the white thread (light) of dawn appears to you distinct from the black thread (darkness of night).

This verse exactly, defines that the Fajr is the meeting point of the nighttime and the daylight, which precisely pinpointed by the arrow. The second one is At-Tur: 49, which reads:

وَمِنَ اللَّيْلِ فَسَبِّحْهُ وَإِدْبَارَ النُّجُومِ

And in the night-time, also glorify His Praises, and at the setting of the stars

Dukes (2011) categorizes the Arabic word *nujuum* as a *genitive masculine plural noun,* which means *stars.* Besides, Duke's inventories show that the word *dabara* appears 44 times in the Koran. In many printed Koran, however, it translates "*idbaro nujuum"* as *in the setting of the stars.* This translation is somewhat misleading, having the fact that there are millions of stars in the sky, which further needs more specific question as to which star is set? Besides, the word to set is naturally only appropriate for a celestial body that has an identifiable size like the moon or the sun. Therefore, it would be more appropriate if we translate *idbaro nujuum* as *the starlight dims due to the presence of the dawn*. Again, the arrow in Figure 1 (left) precisely pinpoints the occurrence timing of the presence of the dawn.

1. 2 Step of the Research

ISRN commenced its research by developing its genuine mathematical algorithm by considering several facts, namely[1]:

i. The SQM collects the physical data. Directly processing such data will be tedious and laborious work because it involves thousands of collected sky brightness data at a temporal resolution of three seconds. In a one full day operation of an SQM, there are approximately 15,000 data points.

ii. Although the plot of the SQM data at any day is unique, they have one characteristic in common. At a certain point, there occurs a point whereby the slope of its moving tangent that initially fluctuates, it then drops consistently becomes negative; This is exactly a point whereby the dawn (Fajr) occurs.

iii. To be free from dealing directly with the physical data, we, therefore, have to be able to approximate the physical data with a mathematical function with which we will be able to simulate the moving tangent along with this function.

iv. Another problem appears as any physical data is not free from stochastic error. This type of errors is advantageous to us because statistics have long been able to characterize its normal distribution. The problem comes into the picture as the SQM data do not only involve the stochastic errors; it also contains changing systematic trends. As such, our selected mathematical function must be able to filter out this systematic trend in such a way that the remaining errors in the physical data are only stochastic (Saksono, 2018).

v. ISRN then chose higher degree polynomials that have the flexibility to filter out such systematic trend. A higher degree polynomial is by no means the only mathematical function that meets the above criteria. Readers are free to use other mathematical functions.

ISRN further developed a quality control system by applying more than ten imaging sensors to verify the presence of the estimated dawn from the SQM data.

1. 3. Algorithmic Development Process

Normally, ISRN takes the SQM data before the official dawn until about sunrise. It is very important because the data must be transparent in such a way that people should be able to identify the point of the presence of the dawn. With a temporal resolution of three seconds, approximately, there are 2,300-2,400 SQM data. Once we can identify its rough position, we

[1] We called it Saksono's algorithm.

then set up an appropriate window within which the polynomial function will likely produce the least root mean square error (RMSE). Eq (1) below gives the general form of the 4th-degree polynomial[2].

$$y_i = p_1 t_i^4 + p_2 t_i^3 + p_3 t_i^2 + p_4 t_i + p_5 \quad (1)$$

where

$y_i =$ *The observed sky brightness data (physical data) at t_i;*
$p_1, p_2, p_3, p_4, p_5 =$ *Parameters of the 4th-degree polynomial;*
$t_i =$ *Time at which the SQM records the data.*

Substituting all data in the selected window into Eq. (1) produces a series of a linear function. Eq. (2) shows them in a matrix format.

$$\begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} t_1^4 & t_1^3 & t_1^2 & t_1^1 & 1 \\ t_2^4 & t_2^3 & t_2^2 & t_2^1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ t_n^4 & t_n^3 & t_n^2 & t_n^1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ p_5 \end{bmatrix} \quad (2)$$

Where:

$e =$ *Vector of the residual error;*
$p =$ *Vector of the parameters of the polynomial to be estimated;*
$A =$ *Design Matrix;*
$y =$ *Vector of the observed value.*

The least-squares approach requires that $\sum_{i=1}^{n} e_i^2$ be minimum. A rather lengthy derivation will end up that we will be able to calculate the parameters of the 4th degree polynomial from Eq. (3) (Mikhail, 1982).

$$p = (A^t A)^{-1} - A^t y \quad (3)$$

Eq. (4) calculates the root means square error.

$$RMSE = \left(\sum_{i=1}^{n} \frac{(y_i - \bar{y}_i)^2}{n - u} \right)^{0.5} \quad (4)$$

where;

$\bar{y}_i =$ The estimated (i.e., calculated) sky brightness data at t_i;
$y_i =$ The observed sky brightness data (the original physical data) at t_i;
$n =$ The number of data in the window;
$u =$ The number of parameters in the polynomial model.

The dawn occurs at the upper-extreme point of the polynomial function as the mathematical model to the physical data. Mathematically, it happens when the following prerequisites hold $\frac{\partial y}{\partial t} = 0$ and $\frac{\partial^2 y}{\partial t^2} < 0$. Algebraically, Eq. (5) and Eq. (6) express these two prerequisites.

$$\frac{\partial y}{\partial t} = 4p_1 t_i^3 + 3p_2 t_i^2 + 2p_3 t_i + p_4 = 0 \quad (5)$$

$$12p_1 t_i^2 + 6p_2 t_i + 2p_3 < 0 \quad (6)$$

Figure 1 shows the use of the 5th-degree polynomial(left) and 4th-degree polynomial (right) for modeling some 700 SQM physical data for June 17 & 18, 2019 in Tacoma, WA, USA.In this

[2] Although, ISRN also developed algorithm for the 3rd and the 5th-degree polynomials.

case, we obtained the RMSE of 0.08 and 0.06 magnitudes, respectively. Furthermore, the calculated dip of -9.7^0 for Fajr and -12.3^o for Isha' in the figure are smaller than the global average of -13.2^0.

2. Verification

Verification is mandatory for any human transaction with and services to the public. It is similar to the principle of *covering both sides* in journalism. A bank teller also has to recheck the amount of money he/she receives before depositing to one's saving-account; however prominent his/her customer is. Although we did not always apply it in all SQM data collection due to convenience and the availability of the proper instrument, we have used more than ten verification systems using a different kind of imaging sensors. They range from an All Sky Camera that is capable of recording half of the whole hemisphere's sky condition, several DSLR cameras ranging from 16 MP to 64 MP geometric resolution, and variety of gadget cameras with lower geometric resolution. Due to memory restrictions, we normally take at a rather coarse temporal resolution of 5-10 seconds.

Even though we have applied various image processing techniques, we have come up with four most favourable image processing techniques, namely, *original (i.e., visual) image, histogram, edge detection*, and *image adjustment analyses*. However, although we are less successful in applying image analysis techniques such as *pixel count*, *image subtraction, a histogram of the adjusted image* and so forth, we might also reuse these image analyses or even other image analysis techniques, in the future.

Having applied a rather high temporal resolution in the image acquisition, it gives us flexibility in analyzing the acquired images at spots appropriate to verify the presence of the dawn or dusk. One should bear in mind, however, after using thousands of images acquired using various imaging platforms, we have concluded that the use of images solely to determine the presence of dawn and dusk is possible although it needs a specific image processing module. In this paper, we only use for the verification process. At the moment, it is special importance as we are in a process to contest hundreds of years-beliefs that the Fajr occurs when the sun is at the depression angle of -20^o.

Figure 1 successfully verifies the presence of the real Fajr calculated from the available SQM data for the same day and location at 5:06:09 on May 12, 2018, in Labuan bajo city, Province of East Nusa Tenggara, Indonesia. Figure 2 also demonstrates the effectiveness of histogram analysis technique to verify the presence of the real SQM Fajr that occurs at 4:40:10.

Figure 1: White spots in the white circle (right) verify the presence of the SQM Fajr (Edge detection of image at 4:33:27 (left) and at 5:06:17 (right) in District of Panguk, Province of Yogyakarta, INDONESIA May 12, 2018)

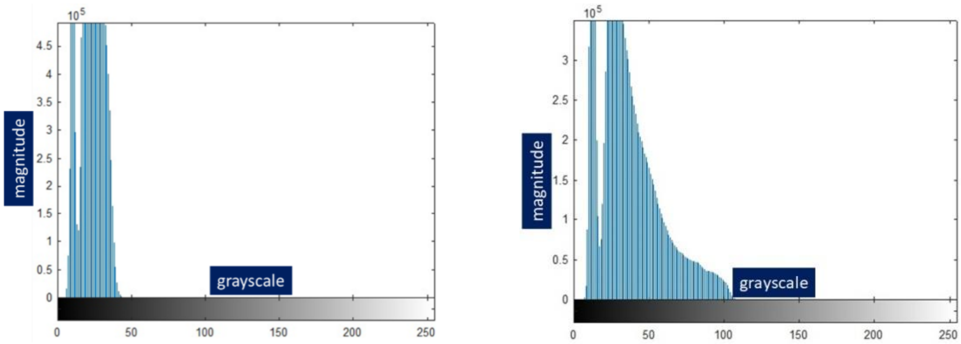

Figure 2: Distinctive difference between the histograms of images at 4:20:50 (left) and 4:41:00 to confirm the presence of the real Fajr according to the SQM data at 4:40:10

3. Results and Their Statistical Measures

Currently, ISRN-UHAMKA has recorded astronomical data in two major region lower and higher latitudes. At the lower latitude around the equator, the so called-Nusantara region, it covers Indonesia and Malaysia, – see Figure 3. Meanwhile, at the higher latitude, the collected data covers Tacoma in Washington State (USA), Cairo (Egypt), and Istanbul (Turkey). Table 1 shows the statistics of the collected data. In spite of limited research budget available, we continue expanding the data coverage, mainly through international collaboration with an institution that has common interests. After data cleaning, for the Nusantara region, we have 257-day data of sky brightnessfor the Fajr, and 171-day data for the Isha'.

Table 1: Statistics of the data collected at lower and higher latitudes.

Statistics	Indonesia	Johor Bahru, Malaysia	Tacoma, WA, USA	Cairo, Egypt	Istanbul, Turkey
Number of data (days, for Fajr & Isha)	425	3	23	5	9
Average dip for Fajr	-13.4°	-12.5°	-11.0°	-13.1°	-9.8°
Average dip for Isha	-13.0°	-10.8°	-12.3°	-	-8.5°
Average Standard Deviation	1.9°	-	1.1°	-	-

Figure 3: Geographical coverage to include 425 data for Fajr and Isha at Nusantara region

From hundreds of recorded and processed data using the SQM, we realized that the dip value at any point on earth turns out to fluctuate throughout the year.

3. 1. The statistical measures of the estimated dip

Because the amount of data in the Nusantara region is far greater than the data in Tacoma, we conducted a more comprehensive statistical analysis of the Nusantara data. For the Fajr, the lowest dip value obtained is -9.4º, and the highest is -18.4º. The average dip (μ) for Fajr is -13.3º with the a-posteriori standard deviation ($\hat{\sigma}$) of 2.0º (see Figure 4). In the meantime, for Isha, the minimum dip is -8.3º, and the maximum dip is -17.7º. The average dip (μ) is -13.2º with the a-posteriori standard deviation of 1.8º (see Figure 5).

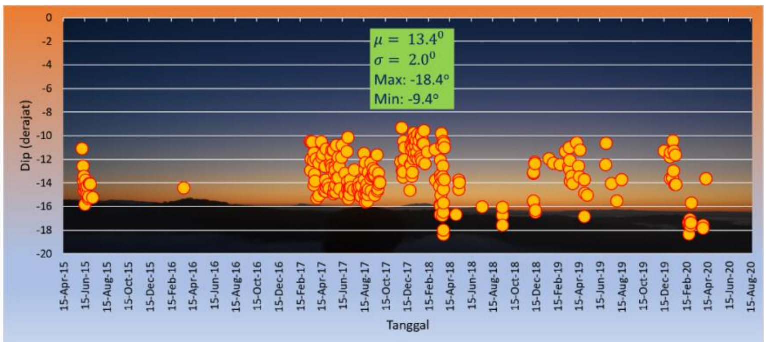

Figure 4: Statistics of the calculated dip from 257 data for the Fajr in Nusantara

These results scientifically confirm some doubts raised by many prominent ulemas back in 2009. Furthermore, the results are apparently in close agreement with the research conducted by the Muslims community in Birmingham, the UK, the so-called Open Fajr Project (Merali, 2016). Open *Fajr* used 42 sets of unobstructed images acquired using an All Sky Camera to obtain the average dip of -13.4º for the *Fajr* in Birmingham.

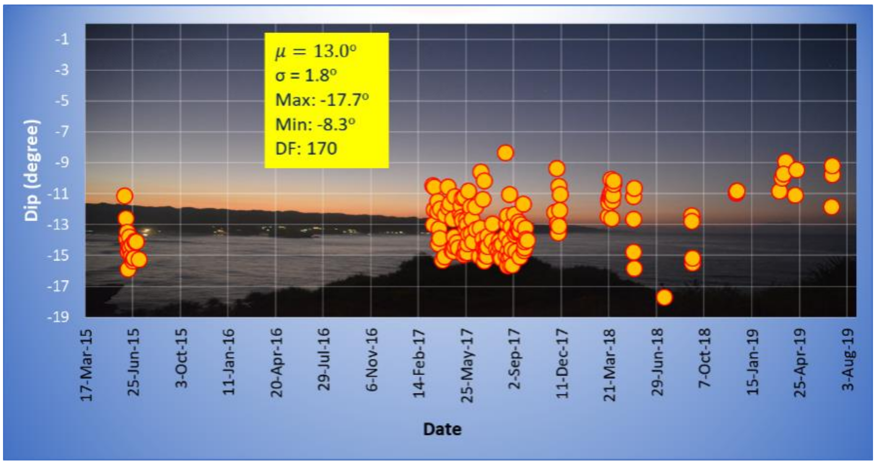

Figure 5: Statistics of the calculated dip from 171 data for the Isha'

3. 2. Reliability of the estimated dip

Baarda (1968) and Gruen (1978) define reliability as: *"the quality of the adjustment model with respect to the detection of model error (i.e., systematic error and blunder)."* In our estimation model, the model error comes from the inappropriateness of our 3rd, 4th, and 5th-degree polynomials to filter out the systematic trend in such a way that the remaining error must be only the stochastic part of the physical data. However, because the average RMSE of the model is within the order of 0.03 magnitudes, one should be confident that the higher degree polynomial model has successfully filtered out the unwanted systematic error.

Furthermore, Gruen (1978) defines the critical value beyond whichis considered a blunder. The upper and the lower limits of such reliability measures are$\mu \mp 3\hat{\sigma}$. From the normal distribution function, statisticians have long been familiar with the 99.9 % (Spiegel, 2000) area within these boundaries. For the estimated dip of the Fajr, the upper and the lower limits are therefore 13.4°$\mp$ 3.16 (2.0°) or 19.7° and 7.1° respectively. Graphically, Figure 4 shows that none of the estimated dips falls beyond this critical point. If we apply this rule-of-thumb criterion to the Isha data, its upper and lower limits are 18.7° and 7.3°, respectively. Likewise, none of the estimated dips for the Isha that falls outside of the upper and lower limits. With these upper and lower limits both for Fajr and Isha', none of the estimated dips must be excluded (see Figure 5). In addition to that, these reliability measures apply:

Any estimated dip for the Fajr, which is larger than 19.7^0 or lower than 7.1^0 is considered blunder. In other words, it is most likely not part of the population due to blunder;

Any estimated dip for the Isha which is larger than 18.7° or lower than 7.3° is considered blunder. Very likely, it is not part of the population due to blunder.

Please bear in mind that the estimated dips in Figure 4 and Figure 5 will retain their stochasticity. Therefore, one might want to apply stricter statistical testing with the criteria to reject the dip that is larger than the certain critical value (Gruen, 1978). For the Fajr, we have:

Critical value:	$\lvert v_i \rvert \geq c(1 - \frac{k_q}{100})\hat{\sigma}_v^o$
Degrees of Freedom:	256
Significance Level:	0.001
Student's t-Table:	3.291
Standard Deviation ($\hat{\sigma}_v^o$):	2.0°

Practically, Gruen (1980) recommends the use of $k_q = 25$ giving $\frac{k_q}{100} = 0.25$. This criterion suggests that we must exclude data with ID#170 for the Fajr. If this happens, out of all 257estimated dips for the Fajr, it is only less 0.4% of the calculated dip that does not meet the above statistical criteria. Likewise, if we apply the above stricter criteria for the estimated dip for the *Ishaa*, we then have the following criteria:

Critical value:	$\lvert v_i \rvert \geq c(1 - \frac{k_q}{100})\hat{\sigma}_v^o$
Degrees of Freedom:	170
Significance Level:	0.001
Student's t-Table:	3.359
Standard Deviation ($\hat{\sigma}_v^o$):	1.8°

therefore, have to exclude ID# 91. Out of 171 data for the Isha', it is only about 0.5%. It proves that, statistically, ISRN's estimated dips for the Fajr and Isha' are highly reliable.

3. 3. Statistical Saturation

If we look closer to the overall dip calculation, we further notice that the average estimated dip has stabilized to certain convergent value. One can check the following Table 2 and Table 3 for the Fajr and Isha', respectively.

These two tables tell us that, statistically, the sample population has been well represented in the whole population. The average dip has stabilized at -13.2°. Adding more data might change the average estimated dip slightly; however, it will not give a significant difference because statistically, it has reached its statistical saturation.

Table 2: The stabilized estimated dip for the Fajr

Date	# of Data	Average Dip for Fajr (degree)	Twilight (minute)	Official Twilight (minute)
Mar 30, 2017	28	13.65	54.6	80
Apr 30, 2017	40	13.45	53.8	80
Jun 30, 2017	70	13.34	53.4	80
Aug 31, 2017	99	13.65	54.6	80
Dec 15, 2018	206	13.34	53.4	80
May 4, 2019	220	13.30	53.2	80
Jul 3, 2019	257	13.43	53.7	80

Table 3: The stabilized estimated dip for the Isha'

Date	# of Data	Average Dip for *Isha* (degree)	Twilight (minute)	Official Twilight (minute)
Mar 30, 2017	28	13.65	54.6	72
Apr 30, 2017	40	13.45	53.8	72
May 30, 2017	59	13.38	53.5	72
Jun 30, 2017	70	13.34	53.4	72
Jul 28, 2017	81	13.44	53.8	72
Aug 29, 2017	99	13.65	54.6	72
Sep 30, 2017	117	13.64	54.6	72
Dec 14, 2018	153	13.27	53.1	72
Apr 18, 2019	159	13.20	52.8	72
Jul 19, 2019	171	13.03	52.1	72

4. Conclusion

The comprehensive summary of ISRN's research concludes that the average value of the presence of the dawn and dusk is when the sun is at -13.4°depression angle for the Fajr and -13.0° for Isha'. It has the consequence that the morning and evening twilights in Indonesia are only in the range of 53-54 minutes. Therefore, the 80-minute twilight for the Fajr Prayer and 72-minute twilight for the Maghrib prayer in Indonesian are way too long. It is because the Indonesian government uses the dip of -20° for the Fajr and -18° for the Isha'. In other words, the current call for the Fajr prayer is about 27 minutes too early, while the call for Isha' prayer is about 19 minutes too late.

Through robust statistical testing, this paper demonstrates that ISRN's estimated dips are highly reliable. Since we have reached the statistical saturation, ISRN-UHAMKA has technically stopped data collection in Indonesia, except for educational and fieldwork for students, university lecturers, researchers, and so forth. We will further extend our focus to scrutinize the pattern of global twilight. Starting in June 2019, we were commencing our mission by sending a team to the US, Europe, and other Asian countries to collect astronomical data. *Inshaa Allah*, this is going to be our contribution to the development of Muslims' civilization.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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